Market Microstructure and High frequency data: Is Market efficiency still a reasonable hypothesis? A survey

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Introduction

Efficiency in market price dynamics is assumed by standard financial theorist. Many asset pricing models in financial literature have been developed in this framework and successively enriched in order to face more complex and appealing problems such as: market risk management, portfolio choices, etc.
According to the market efficient theory (Fama, 1970) markets are populated by homogeneous agents that act in a rational expectation environment where prices fully reflect all the information available; therefore any change in the information should be reflected immediately (by revision the expectations) into price dynamics.
Nevertheless, Grossman and Stiglitz (1980) have discussed the problem of possible information heterogeneity in agents price expectations. They argued that if traders pool information openly a fully reveal equilibrium is reachable, so that people with partial information or without any information could perfectly infer them from prices. Therefore, any possible difference in the agent's information sets could be eliminated by the use of market prices.
Even if an efficient equilibrium can be reached by inference[2], this first attempt to move away from one of the most important assumptions of the classical financial theory has given the basis to the analysis of bounded rationality and heterogeneity of agents.
Distribution of information along traders is not homogeneous, agents have not the same possibilities and capabilities to access to markets and consequently they could form different beliefs about price expectations.
At this point some questions could rise, for instance: is the random walk hypothesis in price changes empirically sustainable? According to the theory, financial asset prices are not predictable and any trading strategies implemented not profitable, is it true in practice?
Therefore we can consider the price formation mechanism as a "black box" where arriving input such as information enter in it, successively these inputs are processed (inside this box) and finally a price is given as output.
In the classical financial theory the mechanism that works inside this box is well known, but when some assumption are relaxed (for instance the introduction of bounded rationality) it becomes not so clear.
Many researchers are involved in answering the previous questions, and consequently to discover this, so called "black box" mechanism; such as "behavioural finance" theorists and econonphysics. The formers try by the use of statistical models and the help of psychology to interpret financial

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[2] In this type of equilibrium we suppose that uninformed agents are able to infer information from prices, so they are "sophisticated agents".

markets by essentially consider agents acting irrationally. On the other hand, econonphysics use the law of physics and natural science to explain financial phenomena.
Therefore, the growing academic and empirical studies on market microstructure, this particular area of financial economics that focuses on price formation and trading processes, have been tried to open up this "black box" using a more flexible and dynamic definition of financial markets.
Nowadays, financial markets operate on high frequency basis. "Tick by tick"[3] data are produced during market activity hours, so the amount of data available, considering that transactions could be temporally separated only by few seconds, is very huge. Thus, this particular environment is a right place for testing the statistical consistency of models and theories but it presents also some problematic.
In fact it appears not so much easy to handle practically such amount of data also because of cost of storage. Development in computer science has notably reduced this problem and, in addition, has profoundly modified the trading process from a completely personal to an electronic one. Nevertheless, personal base trading is not disappeared but still resists as a form of trade.
Furthermore, data are not equally spaced in time, since transactions are not executed in regular time basis but usually time between trades is random. Researchers such as Dacorogna (2001) and Gallo (2006) have tried to face with this problem by interpolations or time aggregation, others have used stochastic processes in order to model time.
The cleanness of data is also very important. Researchers must arrange the data according to the analysis they want to perform, for instance: detect anomalous data, take care of possible wrong ticks or discarding the ones which are not informative and so on. It is not an easy task! Handling database should be done very carefully since statistical properties of models depend strongly on the used data. Brownlees and Gallo (2006) proposes some cleaning procedures.
Even if many studies such as Engle and Russels (1998) and others, have found stylized facts in financial markets such as diurnal or periodic pattern in intraday financial data or strong dependence and autocorrelation in financial returns and volatility clustering, the study of high frequency data should be market related and strongly consider the specific characteristics of the period under study.
Therefore, a careful understanding of the market mechanism considered and the rules governing its participants is of primary interest in order to open up the "black box" of price formation. In addition, since markets are dynamic entities evolving over space and time, rules and mechanisms in markets could be different in the same market during time, or in different markets at the same time.
Although, the use of high frequency data in finance is an important tool for discovering the price formation mechanism in a new "heterogeneous world", the complexity of such an environment could arises new questions that depart from the simple predictability in returns and stimulate new research streams that academics and practitioners should deal with.
In this paper we try, by a survey on the modern microstructure theory and high frequency data literature, to provide to the reader an overview on the most important finding apart from the pure efficient market theory.
This paper is structured as follows: the first session gives an overview of the concept of market efficiency and heterogeneity, while the second session discusses trades and the effect of information, market structure, and the rule of market makers. Afterwards, starting form the simple random walk model for prices we will go through asymmetric and inventory models in the price formation models section, providing in the end a combination of both models. After that, in the fourth session, we will discuss the advantages we encounter in a high frequency environment taking into account also data handling concerns. Stylized facts such as fat tails, temporal dependence, seasonality and persistency are described as source of market inefficiency, in the fifth session.
Then in the last session, econometric tools such as ACD models for durations VAR models for prices and quotas and volatility models in tick time will be stated. Finally conclusions will be argued.

[3] As we shall see in the following, a "tick" is a logical unit of information as a quote or a transaction price.

1. Efficient Market and heterogeneity: an overview

Since the first work of Bachelier (1900) several studies have tried to analyse properties of asset price movements and try to explain their proper characteristics. Lucas (1972) has argued that price movement are consequences of optimal actions of traders in a rational expectation environment were information are not wasted. Especially in equilibrium prices reveal fully information available in the market and any adjustment, due to new information coming has to be reflected properly into prices (Fama, 1970, 1991).

Prices follow a random walk process, therefore any information available for predicting the stock value must be already incorporated into the stock price; the only source of uncertainty is given by the error term. Therefore the price reflected expected values at any point in time and the only source of price change should be unpredictable (random).

Some “behavioural” theorists have started to put some doubts on the efficient market hypothesis simply having a look to mutual fund performance persistency. Burton and Malkiel (1995) have found that managers were able to exploit profits from stock market in a period tend to do so also in the next ones. Some traders could overperform others and the market itself, that is not possible in an efficient word.

Some of them are “searching for alphas” by implementing the so called “Portable Alpha” strategies, where alpha is the measure of a fund portfolio’s risk-adjusted return relative to that of the market, or benchmark.

Along with this empirical finding, some other authors, such as Shiller (1989) try to explain the heterogeneity of agents.

They discriminated between agents present into the market according to their beliefs. The majority of agents do not follow the rational expectation postulates but “fashion” and trends. They have different way to formulate their expectation according to different beliefs, so that the volatility in the market could not only be a matter of unexpected price change, but it can be also “trading generated” and “self generated” (Franch and Roll (1984)).

Namely, uninformed traders could follow an upwards price movement, by buying a particular asset only because they perceive it as a signal of good expectation for that particular stock (they do not have any “hints” about the stock but follow only a “fashion”). The reverse for downwards trends.

Sometime this intuition could be right, other times the upwards or downward trends could be simply do to “noise trader” activity. Camerer and Weigelt (1991) have studied the importance of these phenomena, the so called: “information mirages” and formation of bubbles.

Others authors such as Kurtz (1994), Gouree and Hommes (2000) try to analyse and incorporate in theoretical models, the difference in beliefs formulation and price expectations by using the concept of bounded rationality.

Moreover, the trading time horizon could be another cause of departure from the efficient market hypothesis. Agents could be divided in four categories: intraday traders (traders in the overnight position), daily traders, short-run traders and long run traders (central banks). Even if we suppose that in each class traders are homogeneous in the sense of price expectation formulation and there are not “noise traders”, the market as a whole is going to determine the price of a stock. Many agents with different trade-time horizons and different objectives are interacting and creating an heterogeneous market activity.

These are only some of the theoretical and empirical finding that allows us to depart from the classical financial theory. In the following part of the paper many other findings would suggest us to move towards a new theory of heterogeneous markets, here our scope is to give a flavour to the reader about the context where market microstructure and high frequency data analysis is going to be set.

2. Price Discovery

2.1 Trade and effect of Information

By the use of microstructure theory, financial economists and econometrists try to answer some important questions which have been aroused in the interpretation of modern financial markets such as: What is the rule of markets in the price discovery mechanism? What is the process followed by prices when new information is available? Trading strategies and the market organization as a whole are directly depending on these issues.

Following the classical economic theory, prices denote the expectation of the economic value of assets. In such a prospective, new information could have two possible effects on prices: a permanent effect and a transitional one. The former acts directly on agents' expectations, the latter is related to market frictions. Although trading activity is a sort of "bridge" between information and the possible effects it may have on prices variations, the environmental complexity makes this connection a very difficult object to study.

In general trades can influence both effects. We consider permanent effect first.

When a new information hits the market, stocks could react differently due to the heterogeneity of assets available but also to agents heterogeneity; in fact information sets at their disposal are different and they act with the aim of perceiving different objectives.

This new information could be a private one, namely, it can be an information that is belonging to a particular agent or a set of agents in the market but not available to everyone (asymmetric model are based on this assumption).

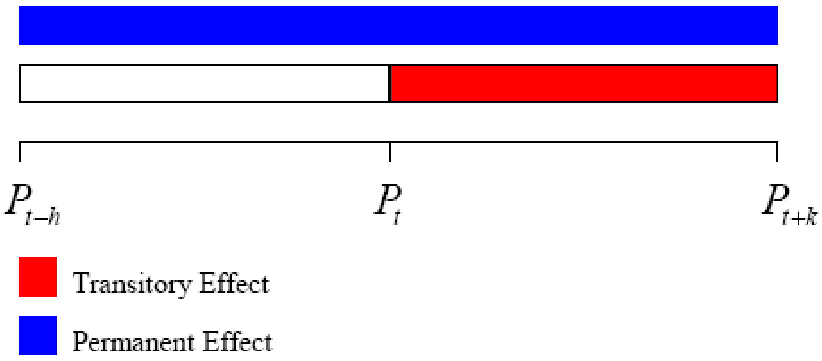
In this context the price movement is a proxy of the private information content of the trade (the buyer is willing to buy an asset when he has a particular information on that stock that the public does not have). Therefore, a change in the permanent component of price movement could be due to the information asymmetry between agents acting in the market.

We should consider also public news: in this case the effect on the price should be equal for every agent but discrepancies on reaction to the news are empirically tested, so also here we have a permanent price movement but it could be not constant for every agent in the same market through time.

On the other hand, the transitory price effect is the perturbation effect of trading activity itself that move away the transaction price from its "fundamental" value, i.e. trading costs.

If we suppose the presence of a unique representative agent in the market and consequently a world where agents are identical and symmetric (also in their information availability), the only permanent effect is due to the impact of public information while the lagged adjustment (that is not contemplate in an efficient world) through it is the transitory effect.

Let us consider p_t the trade price at time *t*, p_{t-h} the pre-trade price: the price of the stock *h* time periods before the trade has occurred, and p_{t+k} post trade price: the price of the stock *k* time periods after the trade has occurred. Therefore, $\gamma = p_{t+k} - p_{t-h}$ measures the permanent component, while $\lambda = p_t - p_{t+k}$ measures the transitory component.



The impact of trade on price is related to market capitalization, trade size and asset type. Precisely, larger trade size and lower market capitalization are usually accompanied with larger price impact of trade. Small cap stocks are more subject to price changing compared to larger ones (Loeb, 1983).
Market liquidity is also a relevant factor. Large trades on large cap stock in liquid market have a very low price impact compared to illiquid market whit the predominance of small caps.
The importance of trading into price formation mechanism has been empirically studied by Franch and Roll (1986). They have noticed that assets return volatility is higher during trading hours rather then no-trading periods. Some explanations have been argued for such a phenomena: many information are arriving during open hours, markets are vehicle of private information to be incorporated into prices by the actions of informed traders and this create volatility and lastly, as we have pointed out in the introduction, trading can create volatility itself. Others authors such as Harris (1986), Jain and Joh (1988) Wood, McInish and Ord (1985) and Wood (1992) have discovered many "anomalies" also in price change during the open hours using intraday data such as U-shaped pattern in bid-ask spread and volatility.
The presence of private information is also a relevant issue. In fact not only new information could move prices, but also beliefs revisions. For instance traders can buy stocks when they think that a stock is underevaluated, this can be derived not only from private news but also from a revision in their personal views.
Moreover, information is not the unique relevant argument in price formation; also the rule of some market operators can be very important in this mechanism. Therefore the "price setters" or Market Makers are crucial.
Before a discussion on their rule, some clarification on the functioning of market variables has to be done. The next paragraph we will deal with this issue.

2.2 Market Structure

Although the structural forms of financial markets are different around the world both spatially and temporally, here we are going to illustrate some market data which are used commonly everywhere.
Quotes are particular data that contain information about the best conditions of trading in the exchange. These information are: quote time stamp (date and time of order execution), bid price (price, for a single share of asset, at which it is going to be sold), bid volume (number of lots which are offered), ask price (price of a single share of asset, at which it is going to be bought), ask volume (number of share which are asked). Additional information about the quota condition may be provided. Quotas are set by market makers.
During the trading day, we can have two possible kind of order: market order and limit order.
Market order is an order to buy or sell a certain number of assets at the current price (bid or ask price) while the limit order determines the maximum price at which a trader is willing to buy a

certain number of assets or on in the selling case, it determines the minimum price at which a trader is willing to sell.
In the former case a trader is sure that a transaction will be executed but he is not sure about the timing; in the latter we have the reverse, in particular the possible order execution will be done at the best price. All the orders are ordered respect to time and price in the book. The best limit orders form the bid and the ask quote. Matching between orders generate a trade.
Trades contain information about the executed order. These information are: transaction time stamp (date and time of execution), transaction price (price of a single share of asset exchanged), transaction volume (number of share exchanged).

2.3 Market Makers

Market makers control market liquidity and guarantee continuity on market trade. Difference between bid and ask prices, the so called "bid-ask spread", is a measure of market liquidity: large market liquidity, low spread level and viceversa. Furthermore, in the context of asymmetric information, spreads could be interpreted as a compensation required from the market makers to protect themselves against trades with informed traders. Market makers could learn from traders behaviour so that they are able to acquire a "felling" of the market composition[4] and adjust the spread as a consequence. Some studies on the learning process have been done by Easley and O'Hara (1987).
In the literature we can find two alternatives way to define the rule of market makers in the market: a passive rule or an active one.
In the passive rule view, market makers are considered as "supplier of immediacy". Since they collect all the orders in the book, they can easily find matching between them and consequently trade possibilities. In this context they assure stability and the "bid-ask spread" is considered as return of their service (Demsetz (1968)).
In the active rule, market makers do not care only about continuity and stability but they actively participate to price formation by adjusting them according to their inventory level. Specifically, dealers set prices on the basis of their inventory objectives: if they are long on a particular kind of asset, they will reduce its price in order to attract buyers and reduce their overaccumulation on a specific side of the market; the reverse happens in case of short positions. Therefore market makers do not adjust only spread but they move also prices according with their inventory politics (Smidt (1971)).
Price stability is also guarantee trough an active rule of market makers. For example traders that are willing to buy some share of an asset, do not have to wait for other traders that would like to sell them, but they simply require the dealer service which offers assets through an own stable inventory politics. Hence, the active rule gives an additional scope to the simple order matching provided by the "supplier of immediacy" in the passive rule.

[4] Volume, trade speed, transaction rates are some variables from which market makers can infer market composition. Since informed traders want to exploit their information against the uninformed, higher values in these variables is related with an higher informed trader market composition.

3. Price Formation Models

In this section we will present a review of simple economic model for price formation starting from the simple random walk hypothesis typically used in efficient markets and successively expanding that model in order to take account of asymmetric information (a kind of heterogeneity) and market maker activity (inventory rule in price setting).

3.1 Random walk Price model

Let us consider the following model:

$$p_t = p_{t-1} + \varepsilon_t \tag{3.1}$$

where p_t is the asset price at time *t*, p_{t-1} is the asset price at time *t*-1 and ε_t is the error term. According to the efficient market hypothesis prices behave as a martingale; hence price change are unpredictable. Specifically the structure of error term is usually described by a white noise. We have: $E(\varepsilon_t) = 0$, $E(\varepsilon_t, \varepsilon_j) = 0$ for $t \neq j$ and $Var(\varepsilon_t^2) = \sigma_\varepsilon^2$.
By the properties of martingale processes all the information regarding prices are promptly reported so that the best prediction for the next period price is today price.
In practice prompt transaction reporting could not be always possible. Information costs and proximity to the market are two possible causes of report delay (inefficiencies). Sometime getting information directly into the market is not easy, it could require time, i.e. inquiry are made and successively time is required for getting data. Therefore, some traders, that are not really "near" to the market can get costly information (commission for the service provided) with some delay (Hasbrouck (1995)).
Although we suppose that traders have the same information set, the time required in processing information can vary across agents due to idiosyncratic heterogeneity.
These facts suggest that random walk hypothesis is easily rejected in market microstructure models, hence many researcher would like to understand the degree of divergence as well as give a possible explanation to carry over by modelization.

3.2 Bid-ask spread price model

A simple modification of random walk price process consists in the introduction of bid-ask spread in a "supply of immediacy" market makers context.
First we should split the price into two components: the efficient price component that follow a random walk process and a transitory component that moves away the price from its level of efficiency. We can consider this model:

$$ef_t = ef_{t-1} + \varepsilon_t \tag{3.2}$$

$$p_t = ef_t + s_t \tag{3.3}$$

where *ef* denotes the efficient price and *s* the pricing error. This price error component represents how much, for instance a buyer has to pay in addition to the efficient price. Surely this term has not to be the same across agents, since they are different each other: agent's characteristics have strong effects on the prices they can give and take.

We suppose a constant bid ask spread that is split into two parts, half of it is charged to the efficient price in order to find the bid price, the other half is deducted to the efficient price in order to find the ask price ($q_t^a = ef_t - S/2$ and $q_t^b = ef_t + S/2$). Therefore:

$$s_t = \pm S/2 \tag{3.4}$$

where S is the bid-ask spread.
Accordingly to the side of the transaction the value of s could be positive or negative. We suppose s to be a stationary random process with homoskedastic variance ($Var(s_t^2) = \sigma_s^2$), zero mean ($E(s_t) = 0$), no serial correlation ($E(s_t s_j) = 0$ for $t \neq j$) and not dependencies between increments in efficient price and s ($E(\varepsilon_t, s_j) = 0$ for all t and j).
This last assumption is a crucial one for discriminating between the effect of efficient price variation and the transitory variation in price formulation. In fact smaller is the pricing error variance, closer the price would be to the efficient one.
Roll (1984) rearranged the model in terms of price change as follows:

$$\Delta p_t = \varepsilon_t + s_t - s_{t-1} \tag{3.5}$$

and he has found an expression for the spread S: $S = 2\sqrt{-\gamma_1}$, where $\gamma_1 = E\Delta p_t \Delta p_{t-1}$ is the first-order autocovariance term. The result implies an important finding in terms of stylized facts: the first order negative autocorrelation. We will discuss it in the following sections.
Goldman and Beja (1979) suggest a model where the price adjustment is gradual and, for this reason, the dependencies could be of high order.

3.3 Market Makers price model: the presence of inventory

As we have previously seen, market makers activity and their careful attention to inventory politics have to be taken into consideration into price formation models.
First we consider a modification of a simple random walk model that allows the inventory control by market makers.
Let us consider x_t the signed trade quantity: it is positive if a dealer is going to sell to a trader (so the trader is willing to buy) while it is negative if a dealer is going to buy from a trader (namely the trader is willing to sell to the dealer). Given market makers risk aversion they should control their inventory through time in order to avoid strong position on a particular side of the market[5].
Here we consider the same efficient price equation used in (3.2).
Quotas are directly determined by inventory position and efficient component as follows:

$$q_t = ef_t - bI_{t-1} \tag{3.6}$$

[5] The inventory position at time t is: $I_t = I_0 - \sum_{k=1}^{t-1} x_k$. As time increases, the sum will diverge so that a market maker is over exposed for an infinite amount, an market failure is certain (Gambler's Ruin problem). Hence dealers have to adjust their inventory periodically by moving prices.

where I_{t-1} denotes the inventory position in the previous period and *b* is the intensity coefficient associated with inventory, namely it measures how much the previous inventory stock will affect the quote value today q_t.
Inventory at period *t* is given by the following equation:

$$I_t = I_{t-1} - x_t \tag{3.7}$$

where a positive trade quantity is going to decrease the inventory level since the dealer will sell part of its inventory stock to the trader, the opposite happens in the case of negative trade quantity.
The signed trade quantity is determined partly randomly and partly from the difference between quotas and efficient price. This implies the following formulation:

$$x_t = -a(q_t - ef_t) + \eta_t \tag{3.8}$$

where *a* is a coefficient associated with the difference between quotas and efficient price while η_t is a white noise process uncorrelated with ε_t.
Finally price is the summation of quota and transaction cost (it is a percentage of the signed trade quantity, we call it *k*):

$$p_t = q_t + kx_t \tag{3.9}$$

The complete model is the following:

$$\begin{aligned} ef_t &= ef_{t-1} + \varepsilon_t \\ q_t &= ef_t - bI_{t-1} \\ I_t &= I_{t-1} - x_t \\ x_t &= -a(q_t - ef_t) + \eta_t \\ p_t &= q_t + kx_t \end{aligned} \tag{3.10}$$

From the model description inventory dynamics appear clear. Suppose that traders are willing to buy from the dealer, namely $x_t > 0$, the inventory stock level is going to decrease so the dealer should rise the quota to induce trader to sell and recompose the shortfall.
Empirically some difficulties could rise, in fact inventory data are private information thus it is not easy to find databases on such variable.

3.4 Asymmetric information Price models

Since trade could move prices, one of the most appealing question that should rise is: what does it determine trade? Probably many are the causes but surely informed traders will transmit to prices something about their private information. First we isolate the pure asymmetric effect, later we will study the information and inventory combined together in a single model.
Here the efficient price and the price equations are respectively given by (3.2) and (3.9).
We modify equation (3.3) in the following way in order to not care about, in this first part, for inventory effects:

$$q_t = ef_{t-1} + u_t \tag{3.11}$$

u_t is a simple with noise process. The peculiarity of this model is the error term component of the efficient price equation (3.2). In fact we allow for a particular specification of the error term as follows:

$$\varepsilon_t = u_t + hx_t \tag{3.12}$$

where u_t already appears in equation (3.11) and it reflects the update to the public information set, while hx_t is the information that is contained in the trade. Indeed, *h* is a parameter that reflects how much of private information would be permanently transferred to the price efficient change. We assume that orders arrives randomly i.e. there is not serial correlation on trades. Therefore in that model we are able to split and to evaluate precisely the price impact of private information from the public ones.

Anyways, a researcher should pay attention on the effective meaning of the second term in equation (3.12). Markets in general and researchers in particular are not able to directly infer the private information of agents since their particular nature, so the only possible way for extracting some information of this kind stays in generate believes. This suggests that the value of *h* coefficient could be determined via subjective conjecture.

The complete specification of the asymmetric model is the following:

$$\begin{aligned}
ef_t &= ef_{t-1} + \varepsilon_t \\
\varepsilon_t &= u_t + hx_t \\
q_t &= ef_{t-1} + u_t \\
p_t &= q_t + kx_t
\end{aligned} \tag{3.13}$$

3.5 Asymmetric information and inventory control: a unique model

Let us combining the asymmetric information model with the inventory one.

Here the model is more complex so we will present it and afterwards comment. The model is the following:

$$\begin{aligned}
ef_t &= ef_{t-1} + \varepsilon_t \\
\varepsilon_t &= u_t + h\upsilon_t \\
q_t &= ef_{t-1} + u_t - bI_{t-1} \\
x_t &= -a(q_t - (ef_{t-1} + u_t)) + \upsilon_t \\
I_t &= I_{t-1} - x_t \\
p_t &= q_t + kx_t
\end{aligned} \tag{3.14}$$

Here again we do not modify equation (3.2) but we modify some other components. We study these modification directly by the mean of dynamics: once public information arrives (u_t realizes) q_t would be set and successively trade quantity is determined (x_t). It leads to a transaction price p_t. Finally we obtain a new efficient price that reflects the effect of public (u_t) and trade related news (where υ_t is the trade innovation). The quota takes account of public information and inventory imbalance while the trade quantity considers the difference between quote and efficient price comprehensive of public information.

Ho and Macris (1984) tests inventory models by relating the bid-ask spread with inventory position and they found that these two variables are positively related, namely the bid and ask spread falls when inventory are positive and viceversa. Other researchers such as Glosten and Harris (1988) have found that inventory revision is more statistically significant as the trade size increases.
Hasbrouck (1988) has isolated the effect of inventory politics from information effect and he found that the latter dominates the former. However other researchers such as Manaster and Mann (1996) found stronger inventory effect.
A study of Madhavan and Smith (1993) have tried to stress the inventory effect on price formation by solving a dynamic programming problem. More specifically they allow market makers to actively recompose their inventory stock and, in addition, they permit them to act as active investors. Mainly market makers can adjust their target inventory level through time.
In doing so, the weak inventory effect on prices found by Hasbrouck can be explained as a long-run effect evaluated in a short-run analysis. Therefore it is underevaluated respect to any informational effect. Consequently, the isolation of any possible inventory shift (long-term component) permits to reflect only the short-run effect of inventory control on prices. They have shown that is significant.
Lyons (1995) confirms the result: market makers not only adjust their inventory stock through time but actively participated on trade thus they modify prices.
As we can see the efficient price hypothesis is not an optimal choice in the description of price dynamics. Many other market frictions have to be considered. This departure from the simple efficiency will be more apparent when we will look at statistical properties of returns.

4. High frequency data: Advantages and Handling

In this section we do not treat directly the problem of efficiency in financial market, instead we would like to make the reader conscious about the environment where prices are set. The place and the modalities of data handling are very important, in fact they determine physically the starting point of price discovery mechanism and its regularities.
Most of the financial studies have been published in financial literature deal with low-frequency, regularly spaced data. Mainly two reasons can give an explanation for such a tendency. First, it is difficult to handle and manipulate high-frequency data especially collection storage problems can be very important. Second, in spite of the fact that financial data are coming to market at random time most of the statistical methods are developed in a homogeneous environment thus financial analysis using these methods is a pure artificial derivation from the original market data.
However, the strong development of computer technology and the explosion of internet have strongly reduced the first problem (i.e. some database are published by some internet provider). Operatively, many institutions have hardly increased their investment in computer technology since the price formation mechanism should be looked at in real time. This implies not only a real time data collection and storage but also a prompt updating system able to fulfil all the market agents needs. For instance traders before going into a transaction should have an idea of their current risk position and possibly a conjecture about a possible price movement; only an advanced integrated system can provide such a service.
Even though handling high frequency data can be problematic and time-consuming, these databases have some positive peculiar characteristics that are not always easy to find.
An extreme amount of data available is a very strong statistical advantage. High precision parameters estimates and rousted measure of volatility can be performed even if we are analysing short interval of time. Most importantly we can distinguish, with an high degree of accuracy, the correct data generating process among many possible ones, so that identification problem may be reduced.

Furthermore, models implemented in high frequency framework allow for more complexity structure. Non linear models with many parameters to estimate are not usually a big problem since the huge availability of data guarantees sufficient degrees of freedom.
Sometime this flexibility is not a common feature of low frequency models, hence many researchers try to extend their sample in order to get more data even if structural change problems can be detected. Precisely, as the time interval dimension augments the structural characteristics of the system under study could vary, so availability of huge amount of data in short time periods allow us to consistent validate statistical models avoiding structural brakes.
Lastly, high frequency databases open the doors for time scale analysis. Some empirical properties are similar at different time scale: behaviours that are common in short interval time are also reencountered, by the use of appropriate aggregation methods i.e. scaling laws, in long run analysis. First studies on the properties of scaling laws were done by Mandelbrot (1963) in a fractal context.
Because data are recorder often, second by second high frequency data are usually called ultra high frequency data stressing the fact that this is the basic level of information.
As we have previously stated, markets are very different each other but they have some common lines. For instance all the centralized exchange collects prices, volumes and information about counterparties involved in a transaction and also time at which it has been executed, with some degree of accuracy. In decentralized markets such as foreign exchange and interbank money market there is not such an automatism, namely “bid-ask spread” are indicatively quoted by banks. While information produced by exchange are collected by exchange themselves, Reuters, Bloomber or other data providers usually operates in decentralized markets collecting data and make them available to agents. Nevertheless these databases are limited in its coverage and in transaction data availability.
Rapid changing in technology and in market laws are common in financial markets so that a full understanding of the market mechanism is required. It follows that all the statistical properties, the handling and construction of time series is heavily related to this fact. Traders should carefully take into account of that when they are cleaning and managing data.
In this section we will explore procedures used for these purposes.

4.1 Data handling

Raw high frequency databases are clearly subjected to errors: filtering these data can eliminate data that does not reflect properly market activity. Errors are more frequent in periods where the trade velocity is high, in fact there is a bigger probability to have mistakes in transferring information when the order flow is very heavy (Falkenberry (2002)). Quotas are usually less accurate then trade; two could be the explanations, first quotas are more then trades, second there is more accuracy on trade since they formally states the result of a transaction.
Dacorogna (2001) has proposed an algorithm to discard wrong observations in an exchange rate markets, Brownlees and Gallo (2005) proposes another kind of procedure in order to eliminate outliers. They evaluate for instance an observation taking into account its distance from the neighbourhood observations. More precisely, they calculate the simple mean and standard deviation of the neighbourhood *k* observations around that one under analysis; they also take into account a granularity parameter. Its rule is crucial, in fact sometime price does not change during some periods so the variance for these subsequent prices is zero, thus a positive lower bound on price variation is required. If an observation is inside the bound (it is given by three times the sample variance plus the granularity parameter) it is kept, otherwise it is considered as outlier, thus discarded.
Once data cleaning is done, researchers should pay attention to correctly manage the data.

At the same time many transaction could happen also at different price, so which is the true price to keep track into the database? Before answering this question we should understand which may be the causes of such a phenomena.
First, securities could be exchanged in different exchanges, second an execution can produce more then one transaction report and third some approximation might happen.
In literature some aggregation methods have been proposed for dealing with this problem: take the median transaction price is a possible solution. For volume or transaction counts simultaneous observation are substituted with the sum of simultaneous volumes or the number of simultaneous transactions.
As we shall see in the section dedicated to the importance of time, high frequency data are irregular spaced date where two subsequent observations are separated by a random interval of time. Although we can lose some important information, in many cases it results more convenient to work with equally space time intervals, hence the choose of an appropriate aggregation method is an important tool for obtaining a lower frequency time scale.
In the literature it has been proposed the use of aggregation functions (Brownlees, Gallo (2006)) or interpolation methods (Dacorogna (2001)). The former methods are based on the use of information in a time interval. Specifically, if we want to lower the frequency of time series and we are interesting to resume all the data in the interval time from *j*-1 to *j* by a unique observation, say y_j, they propose some methodologies such as: the first (last) observation method where the first (last) time interval observation, namely the one observed at time *j*-1 (*j*), is the representative for all the interval; minimum (maximum) method where the minimum (maximum) trade in the interval is the representative one; the sum method where the sum of all the trades happen in the interval is going to be the representative.
Of course, aggregation methods could loose some important statistical features of time series, but in some cases (for example in the aggregation of volumes in simultaneous observations and in the construction of empirical histograms) these methods are required.
Dacorogna (2001) proposed some interpolation methods: previous point, next point and linear point interpolation methods. The availability of dense dataset is important when we apply this kind of methodologies. In fact previous (next) point interpolation work properly in aggregation if it is applied when observations are available (at least in the neighbourhood). If the previous (next) observation is not available researchers should consider the closer to the previous (next) observation as the representative for the missing one. In the case of linear interpolation he should instead use an average between previous and next observations. Here it is very important to have not very distant observations, otherwise we may interpolate data belonging to much different period of time. In this case it is better, in order to reduce the risk of loosing statistical robustness, to not use any interpolation methods but consider the observation as missing one. In liquid stock market there is no problems since a huge availability of data are assured, but in more illiquid market this kind of facts could arise.
Another common problem is the “bid-ask bounce”. Transaction are normally executed at the current bid or ask price even when no news hit the market so, even though no significant events has occurred, prices could move. Price movement inside the so called “bid-ask bounce” are not informative and in some sense misleading for the researcher view point, consequently this changes should be not considered into the analysis. Some algorithms have been constructed in order to detect only price informative movement that are above some thresholds (over the “bid-ask bounce”) and eliminate all the other prices.
Some other problems are related with the opening and the closing time: in fact sometime the official trading day starts a bit later then the official opening and finishes, with the last transaction, also a bit later. In order to take account of that in storing transaction data, researchers usually consider the opening time as the starting point in recording, while they allow five minutes more than the official closing time for assure themselves that the exact closing price is stored.

5. Stylized Facts

Although many financial studies in asset risk management, option pricing (Black and Scholes, 1973) and portfolio theory (for instance in performance analysis Sharpe (1994)) are based on Gaussian asset return distributions and variance as a measure of risk, Mandelbrot (1963) with his seminal work and other researchers (Koedijk (1990)) have stressed the not-Normality hypothesis of financial market. Namely they found a particular regularity in financial asset returns, the so called "fat tail" phenomena.
This is one of the many regularities that have been detected in financial data analysis. In literature these particular phenomena are called "stylized facts".
These findings are crucial in financial modelling, hence researchers and practitioners have to take them into account for avoiding severe risk control problems.
It seems clear that Gaussian world and efficient market hypothesis here are put under critical discussion: in fact the only analysis of two return distribution moments is not enough to describe the intere asset risk profile, but higher moments, such as the fourth one are needed.
In this section we are going to discuss phenomena such as: seasonality, temporal dependencies, volatility clustering and persistence, in addition to excess kurtosis, as important statistical properties typically found in financial data.

5.1 Return asset distribution, "fat tails" and scaling law

In general when we are studying return asset distributions researcher should evaluate empirically the probability of price changes to occur. For doing that, they construct histograms diagrams where probability is measured via empirical frequency. Here it could be useful an appropriate interpolation/aggregation method for obtaining equally space data set.
The distributions obtained are almost symmetric and usually presents very low expected return. The interesting fact is coming from higher moments distribution, such as the forth one. For Gaussian returns excess kurtosis has a zero theoretical value, in financial market instead the kurtosis value are extremely high (especially for short time intervals).
Indeed, the interpolation method could interfere in the proper calculation of that index, but is commonly observed that at higher frequency kurtosis is increasing (Bollerslev and Domowitz (1993)) so that financial returns are not representable by a stable distribution as it is possible in a thin tails Normal world.
This suggests to move our attention from the analysis of central distribution to the behaviour of the tails. Studying the tail is in some sense not a very difficult task in high frequency framework; first because of the huge amount of data available so that tails are very well represented and second because all the possible distribution can be analysed in their tail behaviour only using the tail index (so called α). Hill (1975) proposes a statistical procedure to apply for its estimation.
In general distributions can be divided into three categories according to their tail index value associated:

- thin tail distribution such as the normal one where $\alpha = \infty$ so these distributions have all finite moments and exponential cumulative decay on the tails.
- No tail distribution with $\alpha < 0$
- Fat-tail distribution $\alpha > 0$ with power cumulative decay on the tails[6].

[6] Usually tail index is invariant to aggregation, but we should pay attention when the central limit theorem applies. In fact even if a distribution presents fat tails, its tail index could increase and approach to infinity. Specifically we are going to have few data available in aggregation thus we incur in distortions on the tail index estimation.

Dacorogna (2001) found tail indexes for exchange rates market from 2 to 4, in these cases the fourth moment diverge.
In terms of descriptive analysis, fat-tail and high kurtosis values are an empirical documented fact of non-normality in return distribution. This first finding is in contrast with any efficiency view in the sense of Fama (1970).
Therefore the tail index can be considered as an empirical market efficiency measure: particular values (such as the ones found by Dacorogna) indicate properly the heterogeneous market conditions, where agents react differently to news and the transmission of information is not homogeneous. On the other hand, for higher values of this index normality hypothesis become more realistic even if such values are no encountered empirically, hence we should consider carefully the distorsionary effect of time aggregation.
Another empirical measure of efficiency is given by the helpful introduction of scaling law in finance. As we have previously seen for the tail index we have invariance under aggregation (considering properly distorsions) so, more generally the study of time series should not be dependent on time interval. Thus, it may be an interesting task to understand how different time scale relates each other and possibly define some regularities.
For example Muller (1990), Schnidrig and Wurtz (1995) have found such scaling law in the exchange rate market.
Let us consider a relation between time interval Δt and the power p of the absolute returns in this period:

$$\left[E(r)^p\right]^{1/p} = k(p)\Delta t^{D(p)} \tag{5.1}$$

where *k(p)* is a constant and *D(p)* the drift exponent (according with Madelbrot (1983), (1997)); both are in function of *p*.
For Gaussian random walk for any possible value of *p* we have *D(p)* = 0.5. In exchange markets the drift exponent, for a value of *p* equal to 1, is aproximatively estimated around 0.58. Notice that tail behaviour is captured if we increase the power value. For instance *p* = 2 is more appealing than *p* = 1 in the study of tails.
Anyways, for exchange markets the drift exponent for higher values moves towards Gaussian values, thus also here aggregation play an important role. In this case the study of absolute returns is the right approach to use.
Hurst (1965) characterizes the nature of this exponent and discriminates between uni-scaling or uni-fractal process and multi-fractal process. The former has a constant drift exponent for any value of *p* (the Gaussian random walk is an example) the latter has a drift exponent that depends on *p*.
Di Matteo, Aste and Dacorogna (2003) have studied the scaling law properties of developed financial market by using the Hurst coefficient with a value of *p* = 2. In this case they obtained different drift exponents for different markets. More liquid and developed market such as Nasdaq 100 and Nikkei 225 have values lower than 0.5 while Wig (Poland market) and JSX (Indonesia) present values above 0.5.
Accordingly to this results we can say that the scaling law is not only a measure of efficiency but it is also sensitive to the degree of development of the market. Hence markets are different also in their evolution.

5.2 Temporal Dependence

Large trades should be related to information accuracy. Surely in efficient markets there are no traders that has superior information, but suppose that there exists informed traders that for some extend are rational. More precisely if they know some information they want to exploit as much as they can from their position, thus they should buy/sell massive quantitative of stock.

A theoretical work of Admati and Pfleiderer (1988) has stressed the hiding capability of market agents. Even though they have superior information, they prefer to hide themselves along with uninformed traders so that they are not visible. Hence, big orders are broken up in small medium size ones so that a better price is obtained from the overall operation.
Therefore, price and trade size are related: anyways in an empirical study of Easley large size trade resulted more informative.
Sequence of trades on the same side are the causes of another important stylized fact in finance: positive autocorrelation[7] of absolute returns.
According to the braking order hypothesis researchers have found positive autocorrelation; this phenomena interested prices but also duration[8] and volume.
Muller (1998) has done an autocorrelation analysis for exchange rate market considering absolute returns raised to a *p* power ($|r|^p$). Namely, he tried to make a connection between the extreme events and tail behaviour with autocorrelation properties. He found that as the value of *p* increases, precisely as the importance of extreme events increases, autocorrelation decreases.
This is an important finding: it seems that autocorrelation in returns is principally a matter of central part of the distribution so that extreme events are less correlated each other than average returns.
Another temporal dependence is due to the "bid-ask bounce" we have discussed previously in the section devoted to the data handling concerns: the first order negative autocorrelation. Market makers in a very short-time (for instance few minutes) are biased toward the bid or the ask price since they are willing to balance their inventory positions (Market makers bias). This effect is an important one, since it leads to measurement errors that has to be considered for instance when scaling laws are applied and drift coefficient estimated[9].
These two stylized facts contribute to the view of heterogeneous behaviour of agents in markets. Even though we allow agents to have some bounded rational behaviour in choosing their order flow, they are mostly responsible for positive autocorrelation in returns.

5.3 Seasonality, persistence and Volatility

The autocorrelation analysis has given also important hints in finding seasonality. Some positive autocorrelation has been detected around 1 day lag and 1 week for absolute returns, similar pattern is obtained for squared absolute returns while returns themselves are uncorrelated.
Furthermore, the autocorrelation present in financial market can be well modelled by using GARCH models (Bollerslev, 1986). Also in finance market microstructure a stylized fact common for low frequency data is present: the so called volatility clustering. High volatility periods due to large price changes tend to be followed by large price changes, the same for low volatility periods.
Although GARCH models are implied in the modelization of conditional heteroskedasticity, their application in an high frequency context could lead to some problems. First, if we consider the sum of coefficients it will be less and near to one (so a stationary GARCH process with high persistency in volatility clustering) we should expect an exponential decline in the autocorrelation when there is a volatility shock. However, financial data exhibits a much slower decay effect, so that Baillie, Bollerslev and Mikkelsen (1996) have proposed a model that assigns hyperbolic weight to the previous history of the process: FIGARCH models.

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[7] We are interested in autocorrelation function when we want to analyse linear dependencies between current and past observations.
[8] We are going to discuss more about this variable in the section dedicated to time.
[9] The short-run noise due to uncertainly within the bid-ask spread is accompaigned with long-run bias due to aggregation, so both of them have to be considered in the drift coefficient estimation, otherwise we could have, for instance in the first case, as a matter of noise, different drift exponent estimations. Namely, we can come up with a drift coefficient that is dependent to the short-time interval analysed even if we are considering data coming from the same data generation process. Our goal instead is to have the same coefficient in every short or long time interval.

Second, spurious GARCH coefficients are coming from data that exhibits seasonality.
Many adjustment methods in econometric literature have been proposed for taking account of this last fact and depurate time series: seasonal dummies (Baillie and Bollerslev, 1990), Fourier transform (Andersen and Bollerslev, (1994)) and time-scaling (Dacorogna, (1993)).
Here we concentrate on the last one with the aim to detect some other stylized facts that can enrich our view of inefficiency and heterogeneity in markets.
Dacorogna has concentrated his analysis on exchange rate markets and precisely he has constructed a new time scale, the so called θ-time scale. Considering that global activity of exchange rate markets as a cumulation of local market activity done in single geographical areas, he constructed a time scale structure where empirical local seasonality were carefully modelled, so that only persistent effects can survive. He has related time to market activity (measured via volatility): he expanded the duration of the day when the level of market activity is high (high volatility) and reduced the duration of the day when the level of market activity is low, so that higher volatile days are longer than lower volatile days. He has given appropriate weights (according to market activity) to daily hours[10].
In this "new world" even though the seasonality previously described are eliminated, other picks in the autocorrelation function for the absolute returns are found. These phenomena are related to persistency ("meteor shower hypothesis" found by Engle (1990) and "heat wave" effect). Furthermore, it is very clear the hyperbolic decay of autocorrelation so its long memory serial dependence, thus what it has been tempted to be reproduced via FIGARCH is visible with the study of autocorrelation function in θ-time scale.
As we can easily see all these stylized facts are somehow related to volatility. It is strictly related to market activity and it is also an indicator of persistency (clustering). Volatility also has a specific pattern in financial markets, the so called U-shape: volatility is higher at the beginning and at the end of the day.
Therefore considering all the findings and rearrange everything in terms of volatility can be a key way for discussing the heterogeneity in financial markets.
Although we suppose that agents in the market operates according to GARCH structure for volatility, they should have different time constants for the exponential decay in the autocorrelation since for institutional reasons or simply because they have different believes, they are not homogeneous.
Therefore the hyperbolic results is simply an aggregation result. Also in this case efficiency is violated.
According to the efficient market hypothesis volatility should be negatively correlated with market activity since more agents means faster convergence to the "real market price". This is completely in contradiction with the fundamental hypothesis of θ-time scale that indeed is based on empirical finding: more agents means more possibility for traders to decide when it is more convenient to set an operation so volatility that is, in this case trade generated, is positively related with market activity.
Furthermore, we have seen difference in the evolution between markets, but also their geographical location is important. Stronger autocorrelation (so persistency) is detected when the same traders and market are considered (1 day ago) whereas lower autocorrelation is typically found when different operators and different regions are considered (1 and half day ago) This is called "heat wave" effect.
Within a market difference in trade behaviors could also be stressed by using the concepts of fine and coarse volatility. The first considers the mean absolute working day returns averaged over five observations (covering all the working week), while the second is the absolute return over a full weekly interval.

[10] In physical time daily hours are equally weighted. Therefore hours in the weekend have the same weight as hours in much more active market periods of the week (Wednesday or Thursday for instance). This fact can not occur in the new time scale: weekends days are almost ininfluent.

Coarse volatility is responsible for long-run agents behavior. According with their long-run view, long-run traders determine their investment strategy considering trends or, in other words clusters of coarse volatility. Contrary to this approach, short-run traders react to clusters of coarse volatility changing their trading strategy thus they are responsible for the appearance of cluster in the fine volatility. The reverse is not possible, namely long-run agents do not react to volatility caused by short-run traders, simply because they care about the "fundamental" value of an asset.
We can say that coarse volatility predicts fine volatility but not the other way round. This is again another finding in the view of heterogeneity in markets: difference in investment objective determines differences in strategies and different impact on volatility.

6. Econometrics tools

6.1 The importance of time

Differently from canonical macroeconomic studies, in financial markets data are not equally spaced, unless interpolation procedures, time between two transactions, the so called duration, is modeled as a stochastic variable.
In this section we are going to analyze the relationship between time and price formation in order to critically isolate all the findings that validate the hypothesis of heterogeneity in markets.
During some periods, no observations can be registered; in other intervals concentration of trades are observed. This difference in frequency has notable impact in the price mechanism. For example, it is quite reasonable that more information in the market means more trades, so variability of information available during the trading day characterizes the shape of trading and consequently price movements. On the other hand, more trades induce higher volatility and hence stylized facts previously analyzed are commonly encountered.
It has been noticed that more volatility means lower time between transaction and so concentration of observations in very short time intervals, while less volatility corresponds to a relative peace informative thus an increase in durations.
Moreover, if the market regulation imposes short selling constrains, no trade and consequently fall in price might be a results (bad state of the market), hence durations is somehow related not only to volatility and volume but also it is an important variable that carry information on the market state.
Diamond and Verrechia (1987) and Easley and O'Hara (1992) gives the first contributions on the theme of time as a mean of information. In the former contribution informed traders always trade, if they do not trade it means "bad news", in the latter informed traders trade when there is a signal, hence no trade means "no news".
Contrary to an homogeneous prospective of markets, prices move differently in trading and no trading periods because of news arrivals and the trading mechanism itself. Again the analysis of second distribution moment here is helpful, moreover variance values could also vary during the trading day according to the public and private information available to agents (intraday seasonality are commonly observed). Therefore the positive autocorrelation of absolute returns directly responsible of cluster in volatility is revealed when information are available to some agents that exploit them by trading. Consequently, their activity increases market uncertainly and trades become more and more frequent.
Even though this mechanism seems clear, relation between price information volume and duration is not an easy task and it is nowadays a source of many academic debates.
Harris (1986), Richardson and Smith (1994) derive a mixture of distribution models (MODM). They suppose that agents in the markets are different in their risk aversion and market expectations,

so that any traders react to arrival news according to his characteristics. Hence they adjust their reservation price and change consequently market prices.
In MODM the total change in the market price is given by the average change in reservation prices.
It is assumed that price changes are normally distributed. Knowing the number of information arrivals within the day and keeping the number of agents in the market constant, the total price change is simply a mixture of independents normal whit mixing variable the number of information arriving that day. Thanks to this model we are able, starting from an assumption of market agents heterogeneity, to know how price and volume change accordingly to information arrivals.
This model has many appealing characteristics. It is able to capture several stylized fact such as: heteroskedasticty, volatility clustering, kurtosis and positive autocorrelation of absolute returns. Nelson (1990) has found a link between MODM and ARCH literature, namely he has demonstrated that the discrete version of the continuous-time exponential ARCH models can be rewritten as MODM.
Nevertheless, durations is only marginally dressed as a question in this model.
They were able to find a direct connection between price/volume and information while they treat the problem of duration indirectly considering the distribution for arrival of news as the lognormal one[11].
In the literature modelization of time intervals belongs to the class of point processes.
Let us consider a series of strictly increasing random variables, say $t_0, t_1, t_2, ..., t_n, ...$ corresponding to arrivals time, namely when transactions are executed, and let denote with *N(t)* the total number of transaction occurred previously to time *t*. Transaction arrival times jointly considered determine a point process.
A marked point process is, instead determined if at any arrival times more information are provided, these information are called marks such as: volumes, prices, bid-ask spread, etc…
Usually, researchers would like to model the joint probability distribution of next marks and arrival times in order to have an idea when a new transaction could happen and which values the variables under study would assume. This is not easy, although the joint distribution gives the complete characterization of dynamics, empirically it requires an huge computational effort.
Therefore it is useful to extract, from the original joint distribution, only the information that are required for the particular analysis we are dealing with.
Of course, sometimes it could be interesting to know when a new time arrivals is more likely to happen, other times we can concentrate on the value of a particular mark or in the time interval we should wait for the occurrence of an event, etc…
Form this basic knowledge of point processes we can derive the definition of conditional intensity function. Let us consider the following equation:

$$\lambda(t \mid N(t), t_{i-1}, t_{i-2}, ..., t_0) = \lim_{\Delta t \to 0} \frac{P(N(t+\Delta t) > N(t) \mid N(t), t_{N(t)}, t_{N(t)-1}, ..., t_0}{\Delta t} \tag{6.1}$$

It determines the probability of a single event arrival conditional on the total number of arrivals happened till *t* and time arrival realizations from the starting point of the process till *t*. The conditional intensity function is also called hazard function. This will be useful in modeling time duration via ACD models.

[11] This distribution is the one that fits the data better.

6.2 ACD models

Autoregressive conditional duration models (ACD) have been proposed by Engle and Russell (1998) in order to model time between transaction, i.e. durations.
Let us denote duration, say x_i, the difference between two subsequent arrival times $x_i = t_i - t_{i-1}$ and ψ_i the expectation of duration given the past arrival times, namely:

$$E(x_i \mid x_{i-1}, x_{i-2}, ..., x_1) = \psi_i(x_{i-1}, x_{i-2}, ..., x_1) = \psi_i \tag{6.2}$$

duration at time *i* is given as follows:

$$x_i = \psi_i \varepsilon_i \tag{6.3}$$

where ε_i is and i.i.d. process. The baseline hazard function[12] is:

$$\lambda_0 = \frac{p(\varepsilon; \phi)}{S(\varepsilon; \phi)} \tag{6.4}$$

where $S_0 = \int_{\varepsilon}^{\infty} p(u; \phi) du$ is the survivor function. The conditional intensity function for the ACD is given as follows:

$$\lambda(t \mid N(t), t_{i-1}, t_{i-2}, ..., t_0) = \lambda_0 \left(\frac{t - t_{N(t)-1}}{\psi_{N(t)}} \right) \frac{1}{\psi_{N(t)}} \tag{6.5}$$

According with the operational time view as in Dacorogna, the probability of having an arrival news could change over the time so that we might have longer or shorter durations. This is in line with the definition of "time deformation" (Stock (1988)).
In the ACD models Engle and Russell (1998) proceeds according to the following scheme: first they model the expected duration as follows:

$$\psi_i = \gamma + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j} \tag{6.6}$$

here the expected duration depends on *p* duration lags and *q* expected duration lags thus we are modelling ACD(*p*,*q*) and second they establish which distribution density to use for ε_i. They proposed the exponential (EACD) or the Weibull (WACD) where the second one allows for more flexibility. Others extend the possibility choice including Gamma distribution (Lunde, 1998), Burr distribution (Grammig and Maurer (2000)[13].
Therefore, after modelizing the expected duration and choosing the density distribution for ε_i the likelihood function is constructed.

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[12] The hazard function when $t = 0$.
[13] In the literature many other ACD models were proposed, such as augmented ACD, asymmetric power ACD, asymmetric logarithmic ACD, ect. Moreover Zhang, Russell and Tsay (2000) proposes nonlinear ACD models allowing the dynamic of duration expectations to depend in a nonlinear fashion to the previous durations.
A close relation between ACD models and GARCH model is discussed in Hentscheld (1995) and Duan (1997).

Using ACD models for duration allow us for detecting other peculiarity typically founded when we consider heterogeneity in markets.
Furthermore, we can easily show that from (6.3) and (6.6) durations can be modelled as an ARMA(*max*(*p*,*q*),*q*) since the ACD specification is very similar to an ARCH(*p*,*q*) process, hence

$$x_i = \gamma + \sum_{j=1}^{\max(p,q)} (\alpha_j + \beta_j) x_{i-j} - \sum_{j=1}^{q} \beta_j \upsilon_{i-j} + \zeta_i \qquad (6.7)$$

where, following simple calculation, $\upsilon_i \equiv x_i - \psi_i$ is a martingale difference. Considering $\alpha(L)$ and $\beta(L)$ polynomials of order p and q in the lag operator, the simple $\alpha(1)$ and $\beta(1)$ allow us to study the persistency[14]. Also here agents heterogeneity is somehow captured, namely as in model for volatility, we have a strong persistency in the process, even if stationary is guarantee. Therefore, if an informative trader breaks a big order in medium size orders creating the positive autocorrelation of absolute return and volatility clustering, the increasing volatility reduces durations so that trade become more and more frequent, thus persistency of small duration is found.
In addition, typical intraday seasonality effects of volatility are also reflected into the durations such as the inverse U-shape. Also here, it has been proposed some procedures to depurate the process from diurnal patterns, such as two step estimator using cubic splines (Engle and Russell, 1998), flexible Fourier transform with a proportional hazard model (Gerhard and Haustch, 2007) and semiparametric one-step GLM estimator (Veredas, Rodriguez-Poo and Espasa (2001)).
Sometimes researchers are interested in the analysis of arrivals time where the marks take some particular values, the so called thinned point processes.
As an illustration we can consider the case in which durations are determined by arrivals time that move the price above a particular threshold.
In this view we can study the market liquidity using durations (Engle and Lange (2001)): we can look at the cumulative signed volume transacted over a period in which price do not move above a particular threshold. Lee and Ready (1991) proposed an algorithm to compute this cumulative signed volume transacted, the so called VNET. Algorithm computation are based on the size and timing of current and past transaction flows. VNET is a time-varying measure of market depth indicating the total net volume that market is able to absorb before inducing a price change.
Therefore a time variation in expected VNET must result from agents who chose not completely smooth liquidity over time, such as informed-based traders.
As we already know, impatience and rapid trading usually reflects the influx of informed traders when an asymmetric environment is considered. Hence in high transaction rate, high volatility periods, markets are not able to absorb all the volume present before recordering a price change, hence we should have a lower market depth. On the other hand, when market activity is low and we have a certain pace in volatility market depth is higher.
This is to say that market depthness can be interpreted as another empirical finding of heterogeneity of market agents in this case specifically due to market agents information sets.

6.3 VAR Models for Prices and Trades

Here we are interested to evaluate how much a buyer or seller initiated trade would impact in the formation of future expectation prices. Hasbrouck (1991) perform this analysis.
Let us consider Δm_i the change in the mid-price (the midpoint of bid and ask spread) from *i*-1 to *i*, while x_i denotes the signed volume according to Lee and Ready (1991). The following VAR system is specified:

[14] Some restrictions on ACD parameters must be imposed in order to guarantee the positiviness of durations.

$$
\begin{aligned}
\Delta m_i &= \sum_{j=1}^{J} a_j \Delta m_{i-j} + \sum_{j=0}^{J} b_j x_{i-j} + \eta_{1i} \\
x_i &= \sum_{j=1}^{J} c_j \Delta m_{i-j} + \sum_{j=1}^{J} d_j x_{i-j} + \eta_{2i}
\end{aligned}
\tag{6.8}
$$

where the first equation denotes the quota revision and the second the trade.
Many market microstructure frictions allow us to interpret the rule of lagged values in a VAR representation: inventory control effects, price-smoothing effects of market makers against huge price movements, price discreteness (Harris, (1990)).
In the system (6.8) η_{1i} and η_{2i} are the disturbances, namely η_{2i} represent the innovative part of trade, for instance private information. We suppose $E(\eta_{1i})=0$, $E(\eta_{2i})=0$ and $E(\eta_{1i}\eta_{1j})=E(\eta_{2i}\eta_{2j})=E(\eta_{1i}\eta_{2s})=0$ for $s \neq t$.
Estimation of b_j is important: if it is positive buys increase quota revisions while if it is negative sells decreases it. Hasbrouck pointed out the correspondent VMA (Vector Moving Average) representation of VAR and analysed the impulse response function in transaction time.
The result is that the full price impact of transaction is not immediate but is slow, hence it takes some times before the complete realization of it. He also proceeds in a cross sectional analysis in order to relate price impact effect and firm value.
Considering that the impact of a trade on small size firm is larger and more transitory respect to the impact of a trade on a large size firm, first he has standardized price impact measures for firms across his sample. Specifically, he calculates two different measures: the ratio of the price impact of 50^{th} and 90^{th} percentile volume trade over the average price, thus these measures show the absolute importance of the information revealed in trade[15].
Moving from lower to higher market value sub-sample these measures decline so that smaller firms have larger information asymmetries.
These findings stressed the fact that not only prices react not immediately to information issues but also this reaction is transmitted into price changes differently according to the dimension of the firm under analysis.
Moreover, the fact that durations carry information, should be impounded somehow in price change. According with this view Hasbrouck model has been extended by Engle and Dufour (2000). In particular they started from the assumptions that higher trading intensity induces higher price impact of trades, a faster price adjustment to new trade-related information and stronger autocorrelation of trades.
Considering the first equation in the system (6.8) the parameter b_j(also d_j follows the same adjustment) is explicitly characterized as a time varying parameter, so that it is modelled as follows:

$$
b_j = \delta_j + \sum_{k=1}^{K} \vartheta_k D_{j,i-k} + \varphi_i \ln(x_{i-j}) \tag{6.9}
$$

As we can see, the effect of trade on quota revision can be spitted into two parts: the time-of-the day effect and past durations. The intraday periodicities are captured by dummy variables $D_{j,i-k}$, while past duration enters in their logarithmic form. Whatever parameters ϑ_k and φ_i are no zero, b_j

[15] He also pointed out the problem arising from differentiation of long-run variance. In fact two firms could have the same long-run variance but differ in its composition. That is, the proportion regarding specifically trade (depending on private information) respect to the one due to public information could be different. For this reason it could be useful to consider also a measure that account for the trade impact to public information.

assumes fully its time varying nature. Engle and Doufur studied the impulse respond function considering all the system (6.8) with the modification (6.9) an ADL model for arrival time.
They found that time of the day effect is not uniform across assets, for example at the beginning and in the end of the trading day we usually have more information in the market so this fact induce price changes, but the impact on assets is different (we have different estimations of ϑ_k across assets). In addition the φ_i coefficient is negative: longer durations impact less on price change than shorter duration (the impulse response function shifts up/down depending on short/long transaction durations).

6.4 Volatility Modelling in tick time

In the previous part using the VAR approach we study how quota updates given characteristics of a transaction, whereas the focus now is on how characteristics of a transaction affect uncertainly about quota revision. Here, we adapt the well-know GARCH model literature to the high frequency data context the so called UHF-GARCH. The conditional variance per transaction is given by:

$$h_i = \mathrm{var}(r_i \mid x_i, I_{i-1}) \tag{6.10}$$

where r_i denotes returns, while x_i is duration and I_{i-1} is the information set till time *i*-1. Whereas the conditional variance per unit of time is given by:

$$\sigma_i^2 = \mathrm{var}\left(\frac{r_i}{\sqrt{x_i}} \middle| x_i, I_{i-1} \right) \tag{6.11}$$

and $h_i = x_i \sigma_i^2$. For estimating the model a two step quasi-maximum likelihood estimation is proposed. First we should estimate duration via ACD model and secondly σ_i^2 using a GARCH model. However, in the light of possible market asymmetries in agents information sets, researchers should incorporate in expected durations past bid-ask spread and volume as they could cause an increase of volatility (Engle (2000)). However, careful attention should be pay to the temporal aggregation in the model parameters.
Drost and Nijman (1993) tried to take account for this fact using a "weak" GARCH model with time-varying parameters driven by expected trade duration. Anyway, results apply for aggregation from one fixed interval to another, exogenously specified.
Gramming and Wellner (2002) extended ACD-GARCH model framework to model the interaction between volatility and trade intensity. Another possible extension is introduced by the normal duration GARCH process, namely this model is subject to systematic changes in the expected sampling frequency according to how the expected duration in the ACD process compares to one. Volatility affects expected trade durations as well[16].
From this brief review of volatility models we can easily deduce that the problematic issue of differences and discrepancies that characterized financial markets is also encountered in the volatility modelling. Frictions induce researchers to leave the "simple" GARCH model for volatility

[16] Nevertheless Renault and Werker (2008) criticize this approach, namely they stressed the fact that all the dependence takes place through the link between expected duration and instantaneous volatility, though exogenous news events could actually drive both durations and volatility. Moreover they criticize also the structural foundations of that model. A possible solution to address these issues is to use continuous-time stochastic volatility models with random times.

that fit well data in low frequency cases and go further with extension that account for more complicated and in some way more extensive and general form to modelize uncertainly in financial markets.

Conclusions

Classical financial theory considers financial markets a place where homogeneous agents with the same expectations about prices interact each other in a complete rational world. All the news are already incorporated into prices thus no place for profitable trading strategies to be implemented is available.

This theory has been firstly put under discussion by behavioural economists, in fact they found some possible exploitable strategies looking at mutual fund performances. The introduction of ultra high frequency database with enormous amount of data available makes possible the statistical testing of efficient theory directly into markets. This new very challenging context attracts not only economists and econometricians but also scientists coming from the so called "hard sciences" such as physics.

They try to enounce new theories and elaborate new models in order to consider properly the frictions typically present in the markets which are not considered at all in an "homogeneous world". Economists studied carefully activities of different agents like market makers and traders in the markets and with the help of econometricians, they have started to formalize models apart from the simple random walk for prices.

They first allow for a pricing errors component which become the so called "bid-ask spread" when the introduction of market makers is considered, successively they allow market makers to modify prices according to their inventory, after that they introduce information asymmetries recognizing the possibility to have agents more informed than others. Lastly they combine the two approaches and construct a unique model that takes into account both asymmetries and inventory politics.

Although these models where very well formalized in a context where frictions in the markets are rightly considered, we should stress their statistical properties and specifically if they correctly fit the data. Here the vast databases available play, after a correct data management, an important rule in the validation of theories apart from the classical ones.

Researchers detect some regularities on data. They should consider properly these "stylized facts" in the construction and model validation.

Distributions of returns are fat tailed meaning that extreme events should be heavily considered in the risk profile of an asset. Quantitatively, volatility clustering is strictly connected with the fact that traders once they have private information they split the order over a long period of time instead of generation a unique transaction, thus they create autocorrelation in absolute returns.

All these finding move us away from efficiency and make us in a position to think about a possible alternative theory for heterogeneous markets.

Agents are not omniscient; they act according to different objectives, sometime they have different opinions about the "fundamentals" other times the "distance" become a constrain in an optimal allocation activity. In other situation they simply follow "fashion" or act for liquidity needs.

Therefore, different agents with different information set at disposal interact together in markets and the final results are prices.

As an illustration we can make an analogy with physics. Let us consider a pot full of water and consider it as a market. The agents are represented as particles of water.

We can divided the particles of water in four categories according to the distance to the flame, namely the first category particles are the closer to the flame, while the fourth category particles are the one more far away from the flame.

A flame hits the pot, after few minutes according to the intensity of the flame, we should start to see particles nearby the flame to move, they would move faster and faster so that they will hurt other

particles that are a bit far from the flame but near to the first category particles: the second category particles. Successively the second category particles will start moving faster and faster and start hitting the third category particles and so on… This phenomenon could be view in parallel in financial markets. Water particles are traders, the closeness to the flame is determined according to their horizon time namely the closest are the intraday traders while in the fourth category we have the long run traders such as central banks. The flame is the news and its intensity is the measure of importance for that news. Therefore, as news hits the market, some traders react immediately and later on other agents would react consequently. The speedness of reaction strictly depends on the importance of the news: more important news more reaction so that the reaction time reduces ("time deformation" problem). The relaxation time after a shock in the volatility is long because many different agents (with different constant for the exponential decay of autocorrelation in a GARCH approach) are represented in the market. This is very well documented as the hyperbolically decay of autocorrelation function. Furthermore, some other news are private so also asymmetries play an important rule in the transmission of news through prices.
All these finding make us conscious on the market frictions and heterogeneity of agents, but gives also the basis for a new theory of efficient markets.
Even though heterogeneous expectation is a difficult issue to model, researchers should concentrate on finding regularities by the use of these vast databases and start to incorporate them in theoretical models so that market frictions can be reduced and new theories on market agent interaction perceived.
Although a lot of work is done in this sector of finance additional research effort should be done in detecting other possible regularities on data and later use them in proper models.

References

Admati, Anat R., Pfleiderer P. (1988), "A Theory of Intraday Trading Patterns", *Review of Financial Studies*, 1, 3-40.

Andersen T.G., Bollerslev T. (1994), "Intraday Seasonality and Volatility Persistence in Foreign Exchange and Equity Markets".

Bachelier, L. (1900) "Theory of Speculation", *In The Random Character of Stock Prices*.

Baillie R.T., Bollerslev T. (1990), "Intraday and Inter Market Volatility Persistence in Foreign Exchange and Equity Market, *Review of Economic Studies*, 58, 565-585.

Baillie R. T., Bollerslev T., Mikkelsen H.O.(1996), "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 74, 3-30.

Black F., Scholes M. (1973), "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, 81, 637-659.

Bollerslev T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 31, 307-327, 1986.

Bollerslev C. T., Domowitz I. (1993), "Trading Patterns and Prices in the Interbank Foreign Exchange Market", *Journal of Finance*, 48, 1421-1443.

Brownlees C. T., Gallo G. (2006), "Financial Econometric Analysis at Ultra-High Frequency: Data Handling Concerns", *Computational Statistics & Data Analysis*, *Elsevier*, 51(4), 2232-2245.

Burton G. Malkiel (1995), "Returns from Investing in Equity Mutual Funds 1971-1991", *Journal of Finance*, 50, 549-572.

Camerer, C., Weigelt, K. (1991). "Information Mirages in Experimental Asset Markets", *Journal of Business,* University of Chicago Press, vol. 64(4), 463-93.

Dacorogna M.M. Gencay R., Muller U.A. and Pichet O.V. (2001), "Effective Return, Risk Aversion and Drawdowns", *Physica A*, 289, 66-88

Dacorogna M.M., Muller U.A., Nagler R.J., Olsen R.B., Pichet O.V. (1993), "A Geographical Model for the Daily and Weekly Seasonal Volatility in the FX Market, *Journal of International Money and Finance*, 12, 413-438.

Dacorogna M.M., Pichet O.V., Muller U.A., de Vries C.G. (2001), "Extremal FOREX Returns in Extremely Large Data Set", *Extremes*.

Demsetz, Harold (1968), "The Cost of Tarding", *Quarterly Journal of Economics*, 82, 33-53.

Diamond, Douglas W., Engle R., Verrecchia (1987), "Constraints on Short-selling and Asset Price Adjustment to Private Information", *Journal of Financial Economics*, 18, 1405-1436.

Di Matteo, T., Aste, T., Dacorogna, M.M., "Scaling behaviours in differently developed markets". *Physics A*, 2003, 324, 183–188.

Drost F. C., Nijman T.E. (1993), “Temproal Aggregation of GARCH Processes”, *Econometrica*, 61, 909-927.

Duan J.-C. (1997), “Augmented GARCH(p,q) process and its diffusion limit”, *Journal of Econometrics* 79: 97 127.

Easley. D., O’Hara M. (1987), “Price, Trade Size and Information in Securities Market”, *Journal of Financial Econometrics*, 19, 112-138.

Easley D., O’Hara M. (1992), “Time and the Process of Security Price Adjustment”, *Journal of Finance* 47, 577–605.

Engle R. (2000), “The Econometrics of Ultra-High Frequency Data”, *Econometrica*, 68, 1, 1-22.

Engle R., Dufour (2000), “Time and the Price Impact of a Trade”, *Journal of Finance*, 55, 2467-2498.

Engle R., Ito T., Lin W.-L. (1990), “Meteor showers of Heat Waves?” Heteroskedastic Intra-daily Volatility in the Foreign Exchange Market”, *Econometrica*, 58, 525-542.

Engle R., Lange J. (2001), “Predicting VNET; A Model of the Dynamics of Market Depth”, *Journal of Financial Markets*, V4N2, 113-142.

Engle R., Russell j. (1998), “Autoregressive Conditional Duration: A New Model for Irregularly Spaced Data”, *Econometrica*, 66, 5, 1127-1162.

Falkenberry T. N. (2002), “High Frequency Data Filtering”, *Technical Report*, Tick Data.

Fama E.F. (1970) “Efficient Capital Markets: A Review of Theory and Empirical Work”, *Journal of Finance*, pp. 383-417.

Fama E.F. (1991), “Efficient Capital Market: II”, *Journal of Finance*, 46, 1575-1617.

French K., Roll R. (1984), “Is trading self-generating?” *Working Paper no. 121. Chicago*: University of Chicago, Centre for Research in Security Prices, February.

French K., Roll R. (1986), “Stock Return Variances: The Arrival of Information and the Reaction of Traders”, *Journal of Financial Economics*, 17, 5-26.

Gerhard F., Hautsch N., (2007). “A Dynamic Semiparametric Proportional Hazard Model”. *Studies in Nonlinear Dynamics & Econometrics*, Berkeley Electronic Press, 11(2), 1377-1377.

Glosten L. R., Harris L. (1988), “Estimating the Components of the Bid-Ask Spread”, *Journal of Financial Economics*, 21, 123-142.

Goldman M., Beja A. (1979), “Market Prices vs. Equilibrium Prices: Returns Variances, Serial Correlation and the Role of the Specialist”, *Journal of Finance*, 34, 595-607.

Gouree C., Hommes C.H. (2000), “Heterogeneous Beliefs and the Nonlinear Coweb Model”, *Journal of Economic Dynamics and Control*, 24, 761-798.

Grammig, J., Maurer K.-O. (2000), "Non-monotonichazard functions and the Autoregressive Condtional Duration Model". *The Econometrics Journal* 3, 16-38.

Grammig J., Wellner M. (2002), "Modelling the interdependence of volatility and inter-transaction duration processes", *Journal of Econometrics*, 106(2), 369-400.

Grossman, Sanford J., Stiglitz, J. E. (1980), "On the impossibility of informationally efficient markets". *American Economic Review* 70:393-408.

Harris L. (1986), "A Transaction Data Study of Weekly and Intradaily Patterns in Stock Returns", *Journal of Financial Economics*, 16, 99-117.

Harris, L. (1990), "Estimation of Stock Price Variances and Serial Covariances from Discrete Observations", *Journal of Financial And Quantitative Analysis* 25, 291-306.

Hasbrouck J. (1995), "One Security, Many Markets: Determining the Contribution to Price Discovery, *Journal of Finance*, 50, 1175-1199.

Hasbrouck J. (1988), "Trades, quotes, inventories, and information". *Journal of Financial Economics* 22, 229-52

Hasbrouck J. (1991), "Measuring the Information Content of Stock Trades", *The Journal of Finance*, 66,1, 179-207.

Hentschel L. (1995), "All in the fmily: nesting symmetric and asymmetric GARCH models", *Journal of Financial Economics*, 39, 71-104.

Hill B. M. (1975), "A Simple General Approach to Inference about Tail of a Distribution", *Annal of Statistics*, 3, 1163-1173.

Ho, Thomas, Macris R. (1984), "Dealer Bid-Ask Quotes and Transaction Prices: An Empirical Study of Some AMEX Options", *Journal of Finance*, 39, 23-45.

Hurst H.E. (1951), "Long-term storage capacity of reservoirs". Trans. Am. Soc. civ. Engrs, 1951, 116, 770–808.

Jain P., Joh H. (1988), "The Dependence Between Hourly Prices and Trading Volume", *Journal of Financial and Quantitative Analysis*, 23, 269–283.

Koedijk K.G., Schafgans M.M.A, De Vries C.G. (1990), "The Tail Index of Exchange Rate Returns", *Journal of International Economics*, 29, 93-108.

Kurtz M. (1994), "Endogenous Economic Fluctuations, Studies in the Theory of Rational Beliefs", 6, *Studies in Economic Theory*. Springer, Berlin.

Lee, Charles, Ready M. (1991), "Inferring Trade Direction from Intraday Data", *The Journal of Finance*, 46, 2., 733-746.

Lucas, R. E., Jr. (1972), "Asset Prices in an Exchange Economy", *Econometrica*, 40, 1429-1444.

Lunde, A. (1998), “A Generalized Gamma Autoregressive Conditional Duration Model”, Working paper Department of Economics, University of Aarhus.

Loeb, Thomas F. (1983), “Trading Cost: The Critical Link between Investment Information and Results, *Financial Analyst Journal*, 39, 39-43.

Lyons R. K. (1995), “Tests of Microstructural Hypothesis in the Foreign Exchange Market”, *Journal of Financial Economics*, 39, 321-351.

Madhavan A., Smidt S. (1993), “An Analysis of Change in Specialist Inventories and Quotations”, *Journal of Finance*, 48, 1595-1628.

Manaster S., Mann S. (1996), “Life in the Pits: Competitive Market Making and Inventory Control”, *Review of Financial Studies*, 9, 953-976.

Mandelbrot B.B. (1963), “The variation of Certain Speculative Prices”, *Journal of Business*, 36, 394-419.

Mandelbrot B.B. (1983), “Fractal Geometry of Nature”, *W.H. Freedman and Company*, New York.

Mandelbrot B.B. (1997), “Fractals and Scaling Laws in Finance”, *Springer* Verlag.

McInish T.H., Wood R.A., 1992, An Analysis of Intraday Patterns in Bid/Ask Spreads for NYSE Stocks, *Journal of Finance*, 47, 753–764.

Muller U. A., Dacorogna M.M., Olsen R.B., Pichet O.V., Schwartz M., Morgenegg C. (1990), “Statistical Study of Foreign Exchange Rates, Empirical Evidence of a Price Change Scaling Law, and Intraday Analysis”, *Journal of Banking and Finance*, 14, 1189-1208.

Muller U.A., Dacorogna M.M., Pichet O.V. (1998), “Heavy Tails in High-Frequency Financial Data”, *Practical Guide to Heavy Tails: Statistical Techniques for Analysis Heavy Tailed Distributions*, R.J. Adler, R.E. Freadman and M.S. Taqqu, 55-77, Boston, MA.

Nelson D.B., (1990). “ARCH models as diffusion approximations”. *Journal of Econometrics*, 45, 7-38.

Renault E., Werker, B. (2008), “Causality effects in return volatility measures with random times”. *Unpublished paper: Department of Economics*, University of North Carolina.

Richardson M., Smith T. (1994). “A Direct Test of the Mixture of Distributions Hypothesis: Measuring the Daily Flow of Information”. *Journal of Financial and Quantitative Analysis*, 29, 101-115.

Roll R. (1984), “A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market”, *The Journal of Finance*, 39, 4, 1127-1139.

Schnidrig R., and Wurtz D. (1995), “Investigation of the Volatility and Autocorrelation Function of the USD/DEM Exchange Rate on Operational Time Scales”, *In Proceedings of the HFDF-I Conference*, Zurich, Switzerland, March 29-31, 1995, 3, 1-19.

Sharpe W. F. (1994), “The Sharpe Ratio”, *Journal of Portfolio Management*, 21, 49-59.

Shiller R.J. (1989), “Market Volatility”, *The MIT Press*, Cambridge, MA.

Smidt S. (1971), “Which Road to an Efficient Stock Market: Free Competition or Regulamented Monopoly?”, *Journal of Financial and Quantitative Analysis*, 14, 837-886.

Stock J.H. (1988), “Estimating Continuous-Time Processes Subject to Time Deformation”, *Journal of the American Statistical Association*, 83, 77-85.

Veredas, D., Rodriguez-Poo, J., Espasa A., (2001), “On the (Intradaily) Seasonality and Dynamics of a Financial Point Process : A Semiparametric Approach”, *Papers 2001-19*, Institut National de la Statistique et des Etudes Economiques.

Wood R. A., McInish T.H., Ord J. K. (1985), “An Investigation of Transaction Data for NYSE Stocks”, *Journal of Finance*, 25, 723-739.

Zhang M., Russell J., Tsay R. (2000), “Information Determinants of Bid and Ask Quotes: Implications for Market Liquidity and Volatility”, Working Paper University of Chicago, Graduate School of Business.